Kennedy
Baptist College

Semester Two Examination, 2020

Question/Answer booklet

MATHEMATICS METHODS UNITS 1&2

Section Two: Calculator-assumed

SOLUTIONS

WA student number: In figures

In words ______________________________

Your name ______________________________

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(5 marks)**

For the events A and B, $P(A) = 0.52$ and $P(B) = 0.25$.

Determine $P(A \cup B)$ when

(a) A and B are mutually exclusive. (1 mark)

Solution
$P(A \cup B) = 0.52 + 0.25$ $= 0.77$
Specific behaviours
✓ correct probability

(b) $P(A \cap \bar{B}) = 0.19$. (1 mark)

Solution
$P(A \cup B) = 0.19 + 0.25$ $= 0.44$
Specific behaviours
✓ correct probability

(c) $P(\bar{A} \cap \bar{B}) = 0.33$. (1 mark)

Solution
$P(A \cup B) = 1 - 0.33$ $= 0.67$
Specific behaviours
✓ correct probability

(d) A and B are independent. (2 marks)

Solution
$P(A \cap B) = 0.52 \times 0.25 = 0.13$ $P(A \cup B) = 0.52 + 0.25 - 0.13$ $= 0.64$
Specific behaviours
✓ $P(A \cap B)$ ✓ correct probability

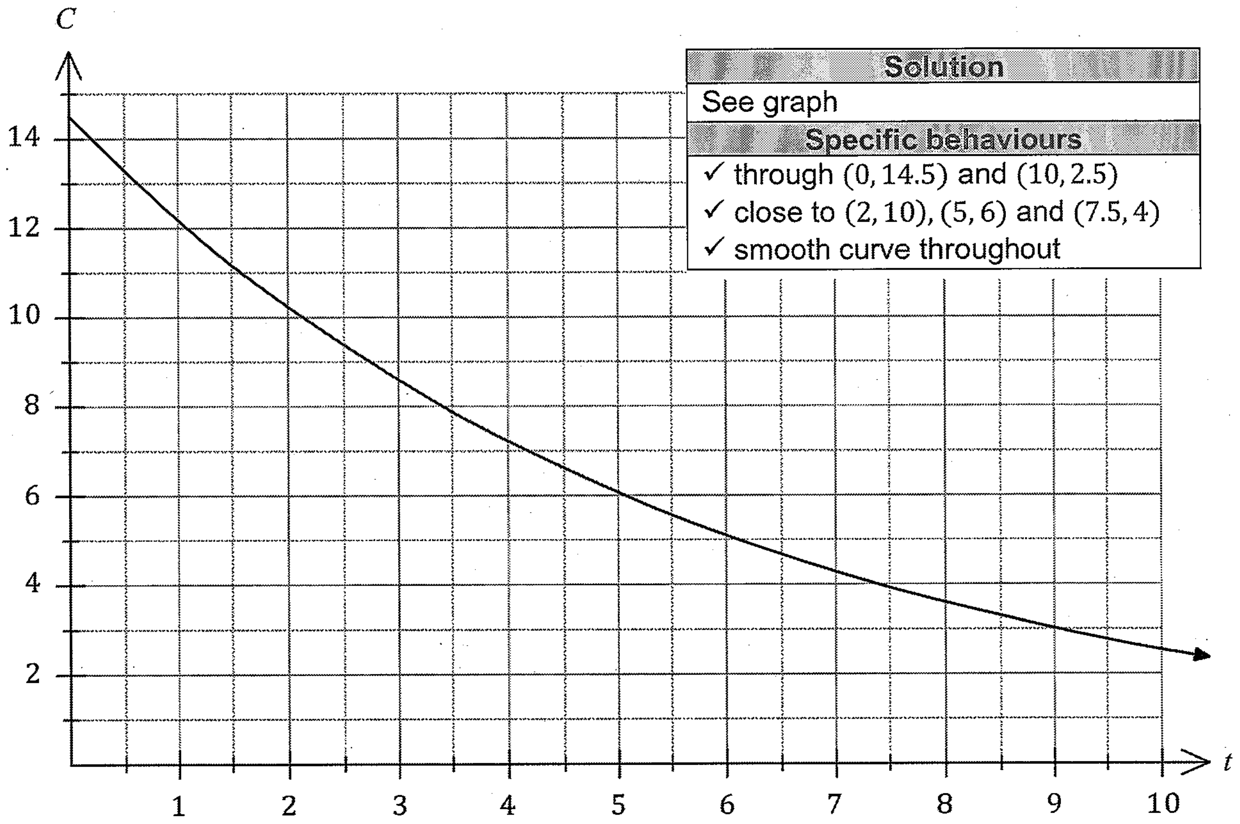
Question 10 **(6 marks)**

The cost, C dollars, for a gigabyte of computer memory between the end of year 2006 ($t = 0$) and the end of year 2016 ($t = 10$) can be modelled by the equation $C = 14.5(0.84)^t$.

(a) Calculate C at the end of year 2010. (1 mark)

Solution
$C(4) = \$7.22$
Specific behaviours
✓ correct cost, to nearest cent

(b) Draw the graph of C against t on the axes below. (3 marks)



Solution
See graph
Specific behaviours
✓ through $(0, 14.5)$ and $(10, 2.5)$
✓ close to $(2, 10)$, $(5, 6)$ and $(7.5, 4)$
✓ smooth curve throughout

(c) Assuming that the model continues to be valid, during which year will the cost of computer memory fall below \$1 per gigabyte? (2 marks)

Solution
$C(t) = 1 \Rightarrow t = 15.3$ Hence during the year $2006 + 16 = 2022$
Specific behaviours
✓ correct value of t ✓ correct year

Question 11 **(7 marks)**

In flat rate depreciation, the value of an asset is depreciated by a fixed amount each year. Using the flat rate model, the value V_n of a machine in dollars after n years is given by $V_{n+1} = V_n - 275$, $V_0 = 3\,850$.

(a) Determine

(i) the value of the machine after 5 years. (1 mark)

Solution
$V_5 = \$2\,475$
Specific behaviours
✓ correct value

(ii) the number of years until the machine has no value. (1 mark)

Solution
$V_n = 0 \Rightarrow n = 14$ years
Specific behaviours
✓ correct number

Using flat rate depreciation, the value of another machine after 5 years will be \$2 695 and after a further 7 years it will become worthless. The value T_n of this machine after n years can be modelled using $T_n = an + b$, where a and b are constants.

(b) Determine the value of a the value of b. (3 marks)

Solution
$2\,695 \div 7 = 385$
$T_0 = 2\,695 + 5 \times 385 = 4\,620$
$T_n = 4\,620 - 385n$
Hence $a = -385$ and $b = 4\,620$.
Specific behaviours
✓ annual loss in value
✓ initial value
✓ clearly states each value

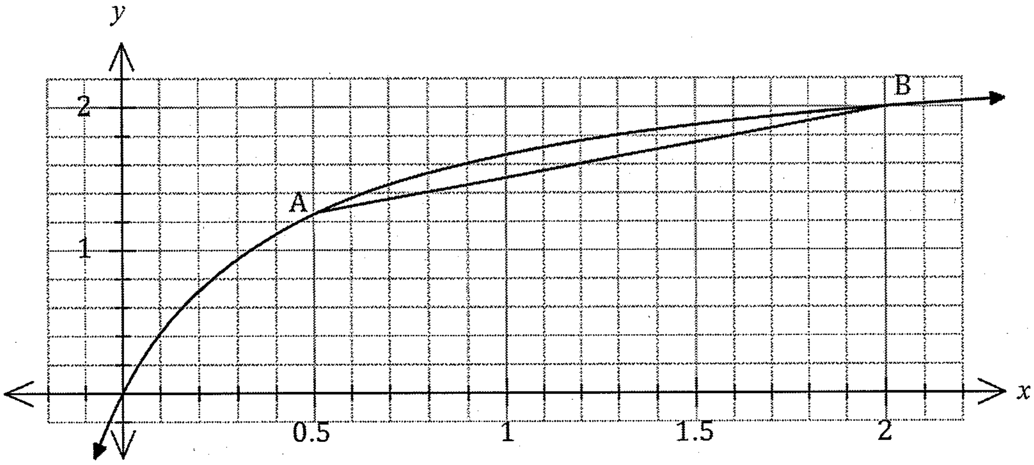
$2695 = 5a + b$ ✓
$0 = 12a + b$ ✓

(c) Given that both machines begin to depreciate at the same time, determine the number of years until the machines have the same value and state what this value is. (2 marks)

Solution
Using a table, the values are both \$1 925 after 7 years.
Specific behaviours
✓ years
✓ value

Question 12 **(8 marks)**

Part of the graph of $y = f(x)$ is shown below, where $f(x) = \frac{5x}{2x+1}$.



Points A and B lie on the curve and have x-coordinates of 0.5 and 2 respectively.

(a) Draw the chord to the curve between A and B on the axes above and determine the gradient of this chord. (3 marks)

Solution
$m = \frac{f(2) - f(0.5)}{2 - 0.5} = \frac{2 - 1.25}{1.5} = 0.5$
Specific behaviours
✓ draws chord on graph ✓ correct y-values ✓ correct gradient

Point C, with an x-coordinate of $0.5 + h$, lies on the curve between A and B. The gradient of the chord AC is m_{AC}.

(b) Calculate m_{AC} for the values of h shown in the table below, recording the gradients in the table to 3 decimal places. (3 marks)

h	1	0.5	0.1	0.05	0.01
m_{AC}	0.625	**0.833**	**1.136**	**1.190**	**1.238**

Solution
See table
Specific behaviours
✓ one correct gradient ✓ at least three correct gradients ✓ all correct gradients

(c) Determine a limiting value for m_{AC} as h becomes very close to 0 and state what feature of the graph of $y = f(x)$ this value represents. (2 marks)

Solution
As $h \to 0$ then $m_{AC} \to 1.25$. This is the gradient of $y = f(x)$ at the point A.
Specific behaviours
✓ limiting value ✓ states gradient at the point A (0.5, 1.25)

Question 13 **(7 marks)**

A set of 175 undergraduates were asked to choose their electives for the following year. 85 chose calculus, 58 chose statistics and 67 chose neither calculus nor statistics.

(a) Determine how many of the undergraduates chose both calculus and statistics. (2 marks)

Solution
$n(C \cup S) = 175 - 67 = 108$ $n(C \cap S) = 85 + 58 - 108 = 35$ Hence 35 chose both electives.
Specific behaviours
✓ indicates union of sets ✓ correct number

(b) Determine the probability that a randomly chosen undergraduate from the set chose

(i) statistics. (1 mark)

Solution
$P(S) = \dfrac{58}{175} \approx 0.3314$
Specific behaviours
✓ correct probability

(ii) statistics but not calculus. (1 mark)

Solution
$P(S \cap \bar{C}) = \dfrac{58 - 35}{175} = \dfrac{23}{175} \approx 0.1314$
Specific behaviours
✓ correct probability

(iii) statistics given that they chose calculus. (1 mark)

Solution
$P(S \mid C) = \dfrac{35}{85} = \dfrac{7}{17} \approx 0.4118$
Specific behaviours
✓ correct probability

(c) Use your answers above to explain whether the choice of statistics and calculus electives is independent for these undergraduates. (2 marks)

Solution
Choice is not independent, as $P(S) \neq P(S \mid C)$. (*Undergraduates are more likely to choose statistics if they have chosen calculus.*)
Specific behaviours
✓ states not independent ✓ explanation using existing probabilities

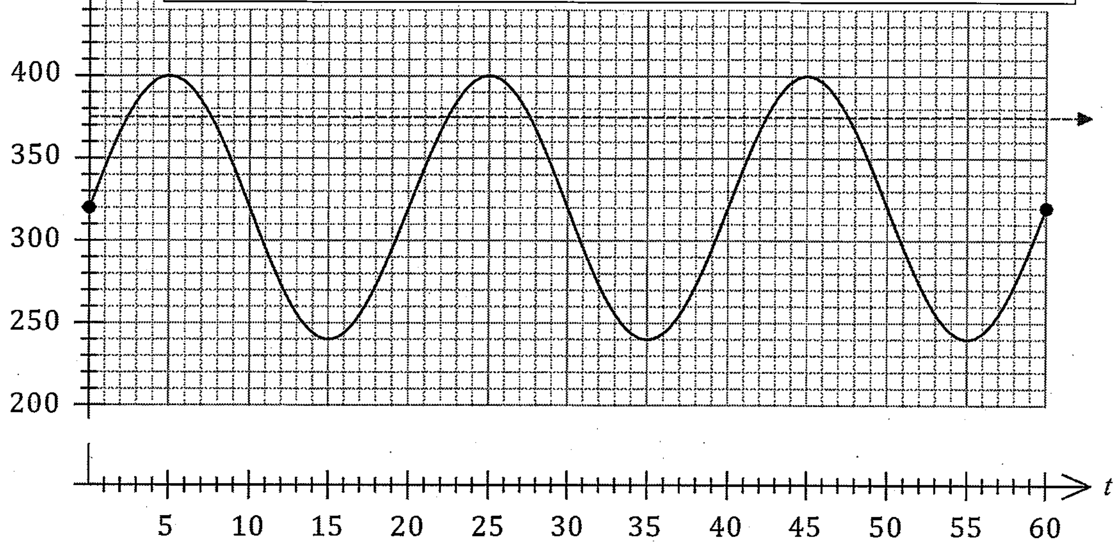
Question 14 **(6 marks)**

When an alternating current is used to power a light globe, the intensity of light emitted from the globe, I lumens, varies with time t milliseconds and can be modelled by the formula

$$I = 320 + 80\sin\left(\frac{\pi t}{10}\right).$$

(a) Draw the graph of I against t on the axes below for $0 \le t \le 60$. (3 marks)

Solution
See graph
Specific behaviours
✓ 3 complete cycles, ✓ locates all min and max, ✓ smooth curve



(b) State the period of I. (1 mark)

Solution
Period is 20 milliseconds
Specific behaviours
✓ correct period

(c) Determine the percentage of each cycle that the intensity of light exceeds 375 lumen. (2 marks)

Solution
$I = 375 \Rightarrow t = 7.587, 2.413$ $7.587 - 2.413 = 5.174$ $\frac{5.174}{20} \times 100 \approx 26\%$
Specific behaviours
✓ indicates interval in ms ✓ correct percentage to nearest whole number

Question 15 **(7 marks)**

A farmer was treating a large area of land for an invasive weed. The area treated on the first day was 275 m^2. Over the following months more resources were utilised so that the area treated each day was 7.5% more than the previous day.

(a) Determine the area treated on the 28^{th} day. (2 marks)

Solution
$T_{28} = 275(1.075)^{(28-1)}$ $= 1\,938\ m^2$
Specific behaviours
✓ indicates use of general term formula ✓ correct area

The cost of the treatment was 35.8 cents per square metre.

(b) On which day did the cost of the days treatment first exceed $10 000? (~~3~~ 2 marks)

Solution
$C_n = 0.358 \times 275(1.075)^{(n-1)}$ $= 98.45(1.075)^{(n-1)}$ $98.45(1.075)^{(n-1)} \geq 10\,000$ $n \geq 65$ On day 65.
Specific behaviours
✓ ~~adjusts sequence~~ ✓ indicates equation/inequality to solve ✓ correct day

(c) Determine, to the nearest ten dollars, the total cost of the first 15 days of treatment. (~~2~~ 3 marks)

Solution
$S_{15} = \dfrac{98.45(1-1.075^{15})}{1-1.075}$ $\approx \$2\,570$
Specific behaviours
✓ indicates use of sum formula ✓ total cost, ~~rounded as required~~ $2571.35

✓ rounded to nearest ten dollars

Question 16 **(8 marks)**

A farm grows two varieties of apples - Fuji and Gala. 42% of all apples are grown in orchard A, 36% in orchard B and the remainder in orchard C. The proportion of Fiji apples that are grown in orchards A, B and C are $25\%, 30\%$ and 35% respectively. After harvesting, the farm stores all the apples together in a large silo before using them to make apple juice.

(a) Determine the probability that an apple chosen at random from the silo is

(i) a Fuji grown in orchard C. (2 marks)

Solution
$P(C) = 1 - 0.42 - 0.36 = 0.22$ $P(C \cap F) = 0.22 \times 0.35$ $= 0.077$
Specific behaviours
✓ proportion grown in C ✓ correct probability

(ii) a Gala. (3 marks)

Solution
$P(A \cap G) = 0.42 \times 0.75 = 0.315$ $P(B \cap G) = 0.36 \times 0.7 = 0.252$ $P(C \cap G) = 0.22 \times 0.65 = 0.143$ $P(G) = 0.315 + 0.252 + 0.143$ $= 0.71$
Specific behaviours
✓ at least one correct proportion   ✓ all correct proportions   ✓ correct probability

(b) Given that an apple selected at random is a Fuji, determine the probability that it was grown in orchard A. (3 marks)

Solution
$P(F) = 1 - 0.71 = 0.29$ $P(A \cap F) = 0.42 \times 0.25 = 0.105$ $P(A\|F) = \frac{0.105}{0.29} = \frac{21}{58} \approx 0.362$
Specific behaviours
✓ $P(F)$ ✓ $P(A \cap F)$ ✓ correct probability

Question 17 **(13 marks)**

A small body is moving in a straight line. Relative to a fixed point O, it has a displacement of x cm at time t seconds given by

$$x(t) = 2t^3 - 28t^2 + 98t - 72, \qquad 0 \le t \le 10.$$

(a) Obtain an expression for the velocity of the body in the form $v(t) = (at + b)(ct + d)$, where a, b, c and d are integer constants. (3 marks)

Solution
$v(t) = \frac{dx}{dt}$ $= 6t^2 - 56t + 98$ $= 2(t-7)(3t-7)$ $= (2t-14)(3t-7)$
Specific behaviours
✓ indicates derivative of $x(t)$ required ✓ correct derivative ✓ factors into required form

Accept $(t-7)(6t-14)$

(b) Determine

(i) the initial velocity of the body. (1 mark)

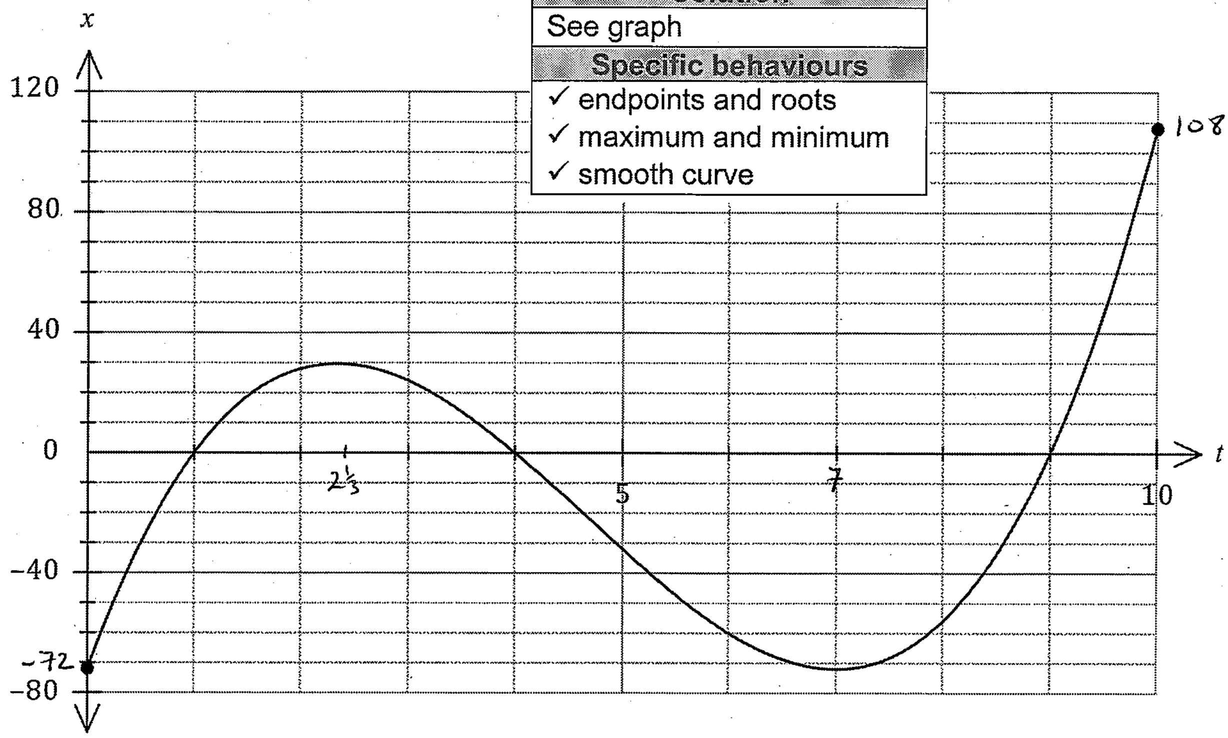
Solution
$v(0) = 98\ \text{cm/s}$
Specific behaviours
✓ correct velocity

(ii) the displacement of the body at the instant(s) that it is stationary. (3 marks)

Solution
$v(t) = 0 \Rightarrow t = \frac{7}{3}, t = 7$ ✓ $x\left(\frac{7}{3}\right) = \frac{800}{27} \approx 29.63\ \text{cm}$ ✓ $x(7) = -72\ \text{cm}$ ✓
Specific behaviours
✓ times when stationary ✓ one correct displacement ✓ both correct, with units

(c) Use the axes below to sketch the displacement of the body over the given domain. (3 marks)

Solution
See graph
Specific behaviours
✓ endpoints and roots
✓ maximum and minimum
✓ smooth curve



(d) State the number of times the body passed through O and determine the minimum speed and maximum speed of the body as it passed through this point. (3 marks)

Solution
Passes through O when $t = 1, 4, 9$ s - on 3 occasions. $v(1) = 48$ $v(4) = -30$ $v(9) = 80$ Hence minimum speed is $30\ \text{cm/s}$ and maximum speed is $80\ \text{cm/s}$.
Specific behaviours
✓ correct number of times ✓ minimum speed (MUST BE +ve) ✓ maximum speed

Question 18 **(9 marks)**

(a) Point $A(11, -5)$ lies on the circumference of a circle with centre $(-4, 3)$. Determine the equation of the circle. (3 marks)

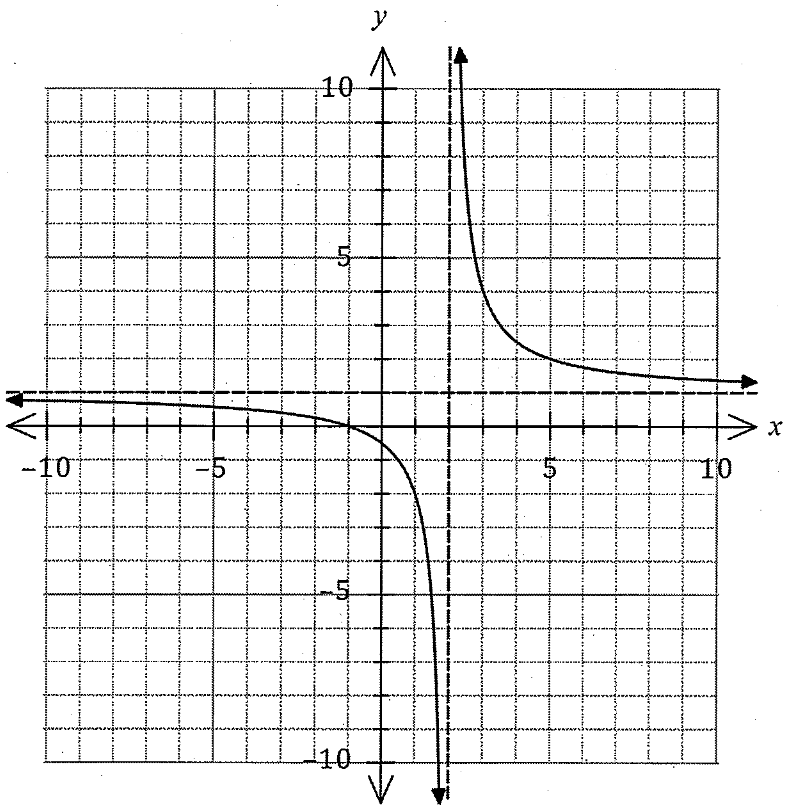
Solution
$(x+4)^2 + (y-3)^2 = k$ Using given point: $15^2 + (-8)^2 = k \Rightarrow k = 289 \ (= 17^2)$ Equation: $(x+4)^2 + (y-3)^2 = 289$
Specific behaviours
✓ forms equation using centre and constant Award ✓ for $(x+4)^2+(y-3)^2$ seen ✓ equation for constant using point ✓ correct equation (any form)

(b) The graph of $y = 1 + \dfrac{a}{x+b}$ passes through the points $(1, -2)$ and $(3, 4)$.

(i) Determine the value of each of the integer constants a and b. (3 marks)

Solution
$-2 = 1 + \dfrac{a}{1+b}, \quad 4 = 1 + \dfrac{a}{3+b}$ Solve simultaneously for $a = 3, b = -2$.
Specific behaviours
✓ uses points to form two equations ✓ solves simultaneously ✓ both values correct

(ii) Draw the graph of $y = 1 + \dfrac{a}{x+b}$ on the axes below, clearly indicating any asymptotes. (3 marks)



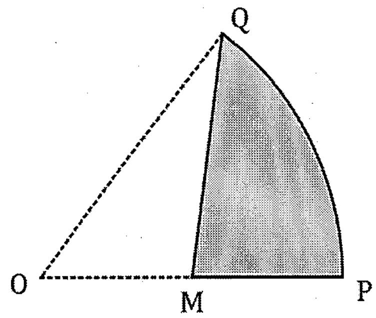
Solution
See graph
Specific behaviours
✓ both asymptotes
✓ LHS, smooth curve, through $(1, -2)$
✓ RHS, smooth curve, through $(3, 4)$

Question 19 **(8 marks)**

The shaded region MPQ in the diagram is a canvas awning and is part of a sector of circle OPQ with centre O and radius 3.8 m.

QM is a straight line from Q to M, the midpoint of OP.

The size of $\angle QOP$ is 1.1 radians.



(a) Determine the area of sector OPQ. (1 mark)

Solution
$A_{OPQ} = 0.5(3.8)^2 \times 1.1 = 7.942\ \text{m}^2$
Specific behaviours
✓ correct sector area

(b) Determine the area of the canvas awning. (3 marks)

Solution
$A_{OMQ} = 0.5(3.8)(1.9)\sin(1.1)$ $= 3.217$ $A_{MPQ} = A_{OPQ} - A_{OMQ}$ $= 7.942 - 3.217$ $= 4.725\ \text{m}^2$
Specific behaviours
✓ indicates use of difference of areas ✓ area of triangle ✓ correct area with units

(c) The edge of the canvas is to be reinforced with thin wire. Determine the length of wire required. (4 marks)

Solution
$QM^2 = 3.8^2 + 1.9^2 - 2(3.8)(1.9)\cos(1.1)$ $QM = 3.391$ $\text{Arc}_{PQ} = 3.8 \times 1.1 = 4.18$ $L = 1.9 + 3.391 + 4.18$ $= 9.47\ \text{m}$
Specific behaviours
✓ uses cosine rule for QM ✓ length of QM ✓ arc length QP ✓ total length with units

Question 20 **(7 marks)**

A reader bought 14 different novels, planning to read a selection of them when on holiday.

(a) Determine the number of different combinations of novels the reader could choose from if they select

(i) six novels. (1 mark)

Solution
$\binom{14}{6} = 3\ 003$ combinations
Specific behaviours
✓ correct number

(ii) five or six novels. (2 marks)

Solution
$\binom{14}{5} + \binom{14}{6} = 2002 + 3003$ $= 5\ 005$ combinations
Specific behaviours
✓ ways to choose five   ✓ correct number

Four of the 14 different novels are by the author Harper.

(b) The reader makes a random selection of six novels. Determine the probability that

(i) none of the novels selected are by Harper. (2 marks)

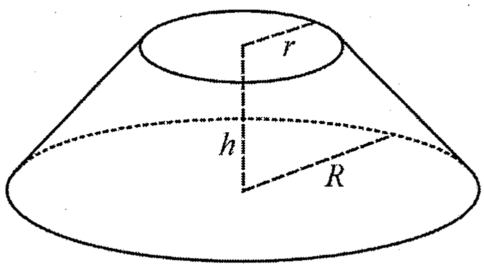
Solution
Must choose from 10 not by Harper: $\binom{10}{6} = 210$ $P = \frac{210}{3003} = \frac{10}{143} \approx 0.0699$
Specific behaviours
✓ ways to choose   ✓ correct probability

(ii) one of the novels selected is by Harper. (2 marks)

Solution
$P = \frac{\binom{10}{5}\binom{4}{1}}{3003} = \frac{252 \times 4}{3003} = \frac{48}{143} \approx 0.3357$
Specific behaviours
✓ indicates correct method   ✓ correct probability

Question 21 **(7 marks)**

The frustum shown at right is a truncated right cone.



The volume of such a solid is $V = \frac{\pi h}{3}(r^2 + R^2 + rR)$, where r is the radius of the smaller circle, R is the radius of the larger circle and h is the perpendicular distance between the two parallel circles.

Consider frustum F where $r = x$ cm, $R = 3r$ and $r + h = 36$ cm.

(a) Show that the volume of frustum F is $156\pi x^2 - \frac{13\pi x^3}{3}$ cm^3. (3 marks)

Solution
$r = x, \quad R = 3x, \quad h = 36 - x$ $\therefore V = \frac{\pi(36-x)}{3}(x^2 + (3x)^2 + x(3x))$ $= \left(12\pi - \frac{\pi x}{3}\right)(13x^2)$ $= 156\pi x^2 - \frac{13\pi x^3}{3}$
Specific behaviours
✓ expresses r, R and h in terms of x ✓ substitutes and simplifies $(r^2 + R^2 + rR)$ term   ✓ clear steps to obtain final expression

(b) Use a calculus method to determine the value of x that maximises the volume of frustum F and state this maximum volume, rounding your answer to the nearest cm^3. (4 marks)

Solution
$\frac{dV}{dx} = 312\pi x - 13\pi x^2$ Derivative is zero when: $\pi x(312 - 13x) = 0$ $x = 0, x = 24$ $V(24) = 29952\pi \approx 94\,097$ Maximum volume of frustum is 94 097 cm^3 when $x = 24$ cm.
Specific behaviours
✓ derivative   ✓ indicates derivative must equal zero   ✓ states root of derivative for maximum volume   ✓ states maximum volume

End of questions